CRS-Updatable Asymmetric Quasi-Adaptive NIZK Arguments

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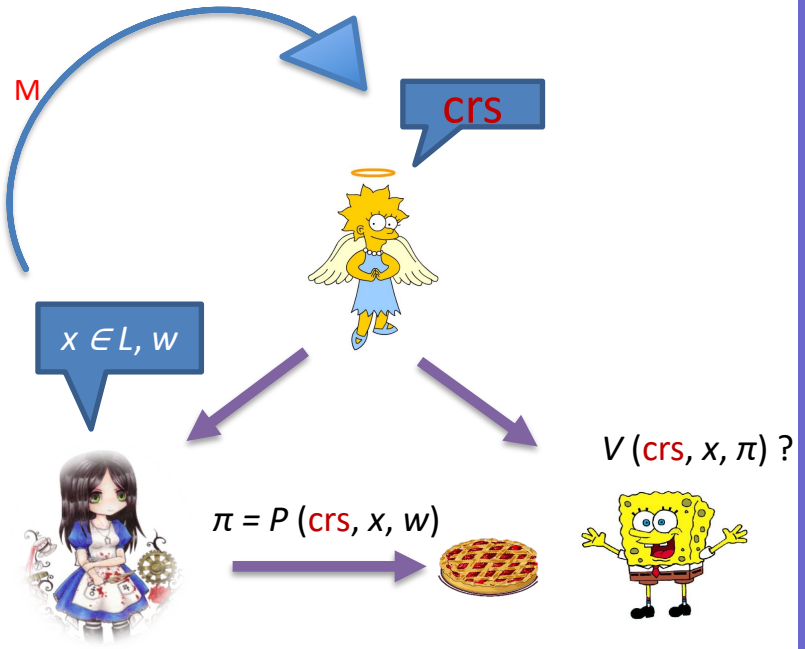
Motivation

- **Quasi-Adaptive NIZK** (QA-NIZK) where the CRS depends to the Language parameter M .
- Such a dependency of the CRS allows one to construct very efficient QA-NIZKs (for linear language) based on standard assumptions.
- QA-NIZK has applications in constructing efficient cryptographic primitives (commitment schemes, IBE, signature schemes, SNARKs compilers, …)



QA-NIZK in CRS model

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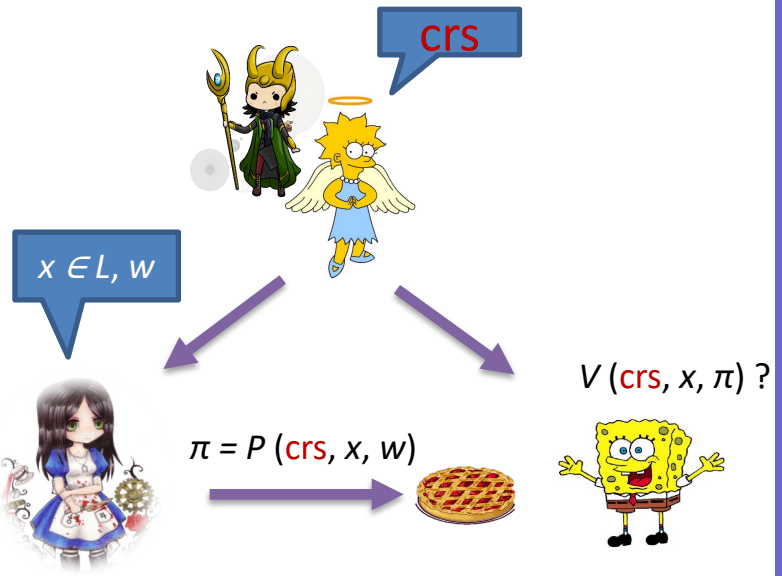


NIZK in CRS model

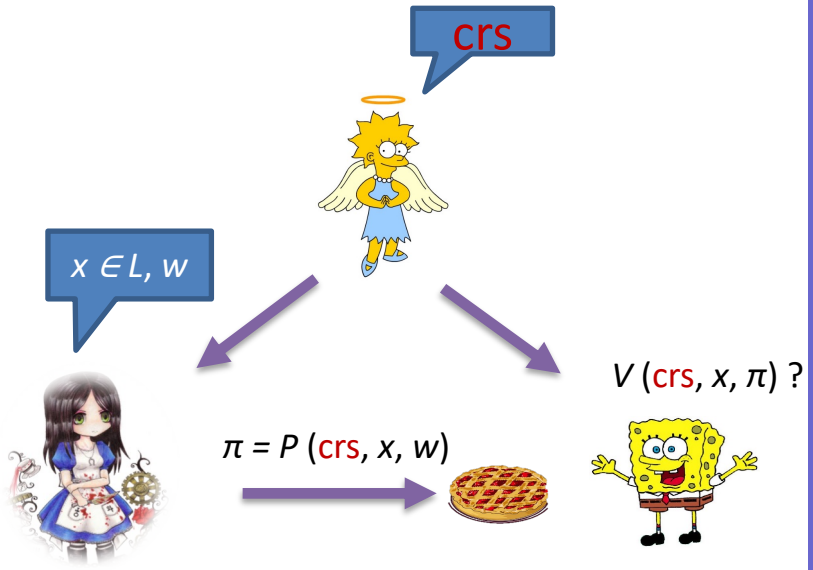
Challenge:

- Such constructions need **a trusted party** to generate the CRS.
  - Is the security guaranteed if the parties do not trust the CRS generator?

Preliminaries:

NIZK in the CRS model

Definition: A NIZK argument system allows the prover to convince the verifier of the validity of some statements and must satisfy the following properties:



NIZK in CRS model

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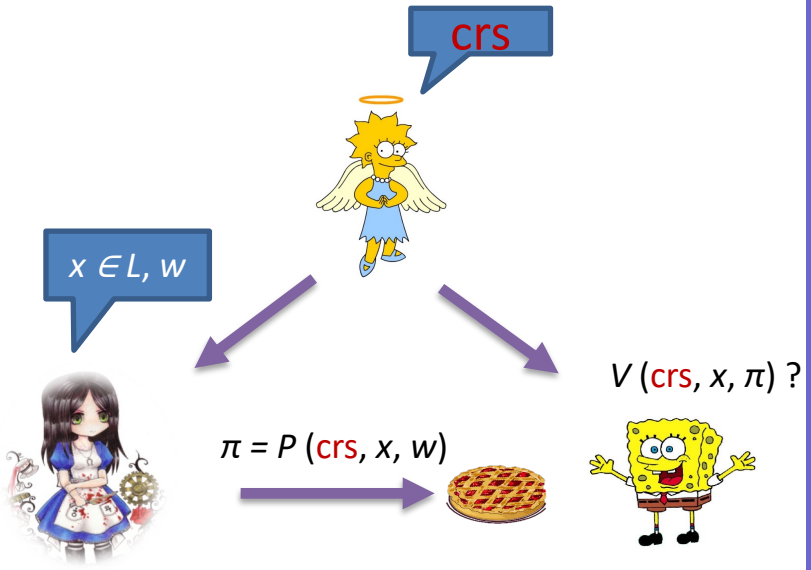
Definition: A NIZK argument system allows the prover to convince the verifier of the validity of some statements and must satisfy the following properties:

Completeness: $x \in L \Rightarrow V \text{ accepts } \pi$

Soundness: $x \notin L \Rightarrow V \text{ rejects } \pi$

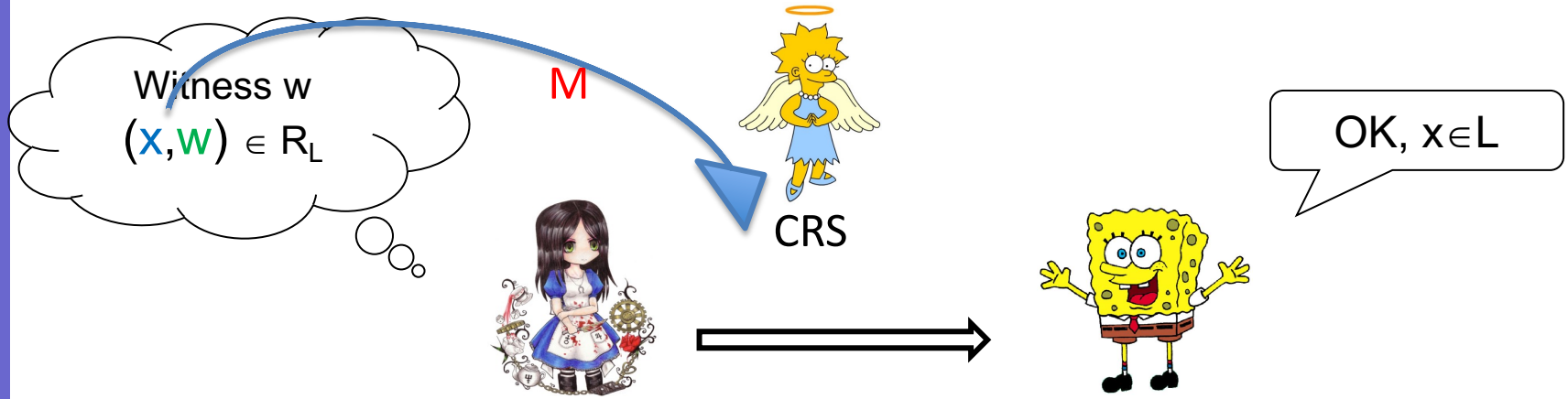
Zero-Knowledge: $\pi \text{ leaks nothing beyond } x \in L$



NIZK in CRS model

- Quasi-Adaptive NIZK
-Asymmetric Quasi-Adaptive NIZK

Quasi-Adaptive NIZK (QA-NIZK)



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- M cannot adaptively depend on the CRS
- Usually, M = some public key

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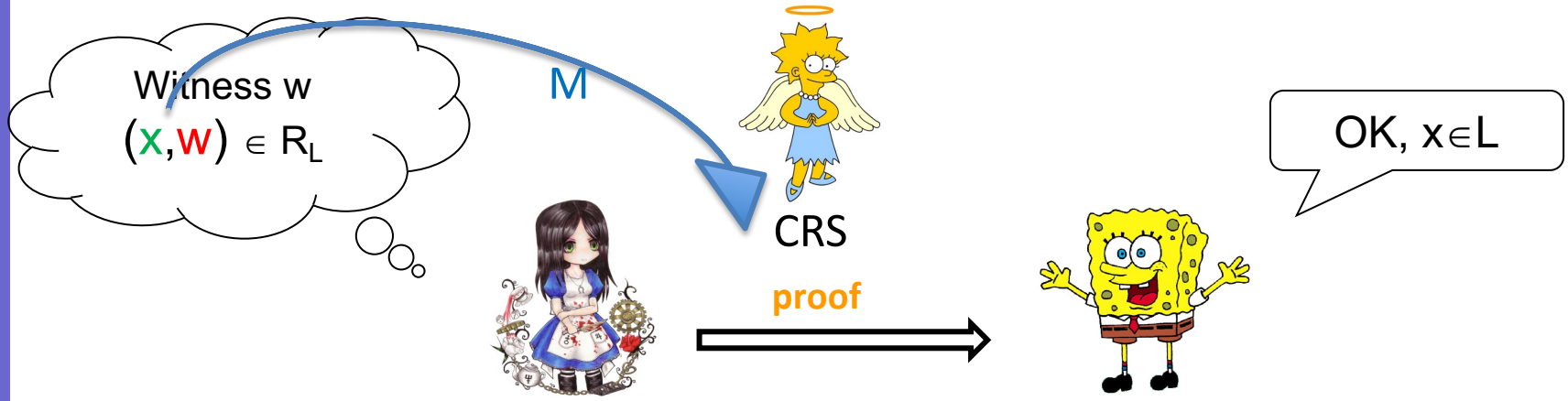

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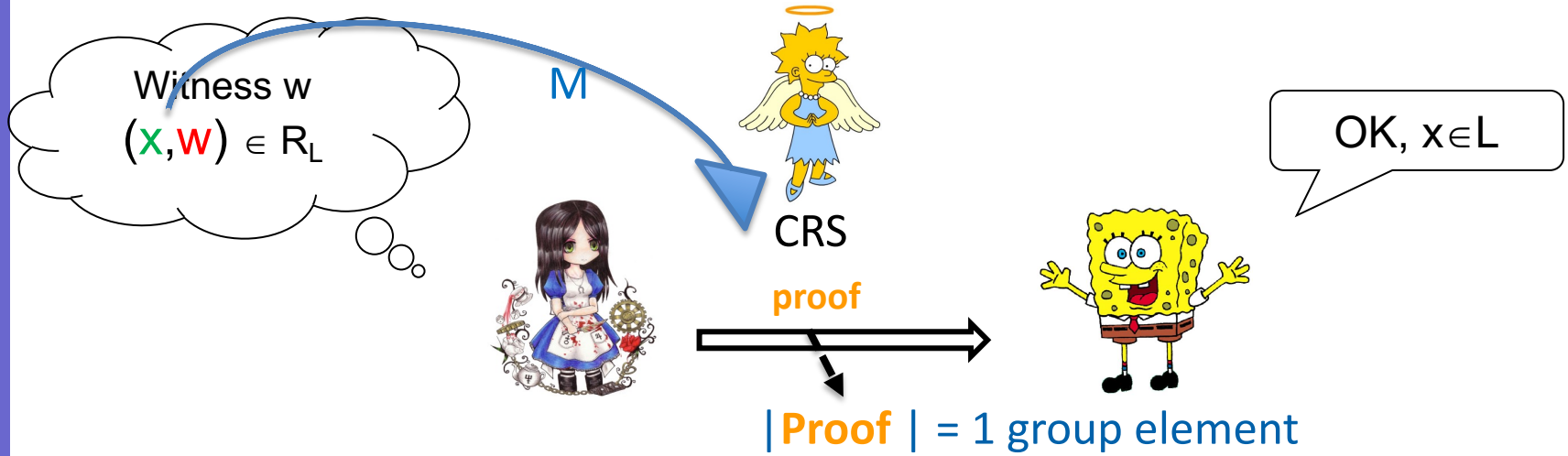

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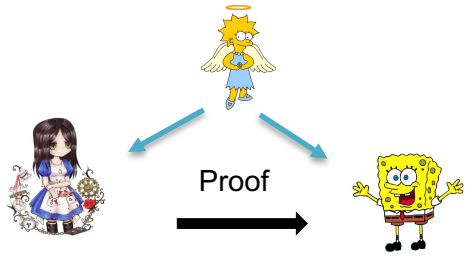
[Roy-Julta Asiacrypt2013] ⇨ [**Roy-Jutla** Crypto2014] ⇨ **[Kiltz-Wee Eurocrypt2015]**

Quasi-Adaptive NIZK (QA-NIZK)

Let G_1, G_2, G_T be additive groups of order p
Denote $[a]_i = ag_i$ where g_i is generator of G_i and $a \in \mathbb{Z}_p$
Assume $\cdot : G_1 \times G_2 \to G_T$ is a bilinear map
$[A]_1[B]_2 = [AB]_T$ for compatible matrices A, B



- [Kiltz-Wee Eurocrypt 2015]
  - most efficient known QA-NIZK for SUBSPACE language
- **Task of QA-NIZK for SUBSPACE:**
  - Fix language parameter $[\mathrm{M}]_1 \in G^{n \times m}$
  - Prove in zero knowledge that $[\vec{y}]_1 = [\mathrm{M}]_1 \vec{w}$ for some $\vec{w} \in \mathbb{Z}_p^m$

$$L = \left\{ [\vec{y}]_1 \in G_1^n \mid \exists \vec{w} \in \mathbb{Z}_p^m \ s.t \ [\vec{y}]_1 = [\mathrm{M}]_1 \vec{w} \right\}$$

Asymmetric Quasi-Adaptive NIZK (QA-NIZK)

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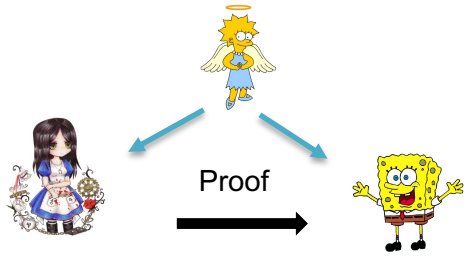
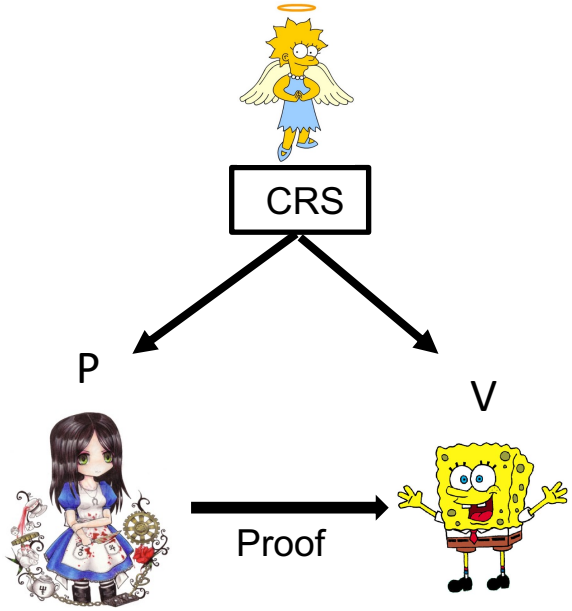


- [González et al. ASIACRYPT 2015]
  - most efficient known asymmetric QA-NIZK for SUBSPACE language

- **Task of Asymmetric QA-NIZK for SUBSPACE:**
  - Fix language parameter $[\mathrm{M}]_1 \in G_1^{n \times m}$ and $[\mathrm{N}]_2 \in G_2^{n \times m}$
  - Prove in zero knowledge that:

$$L = \left\{ [\vec{y}]_1, [\vec{x}]_2 \in G_1^n \times G_2^n \mid \exists \vec{w} \in \mathbb{Z}_p^m \ s.t \ [\vec{y}]_1 = [\mathrm{M}]_1 \vec{w} \wedge [\vec{x}]_2 = [\mathrm{N}]_2 \vec{w} \right\}$$

NIZKs in Different Subversion Model



The CRS Model
(i.e., [EC:Groth16])

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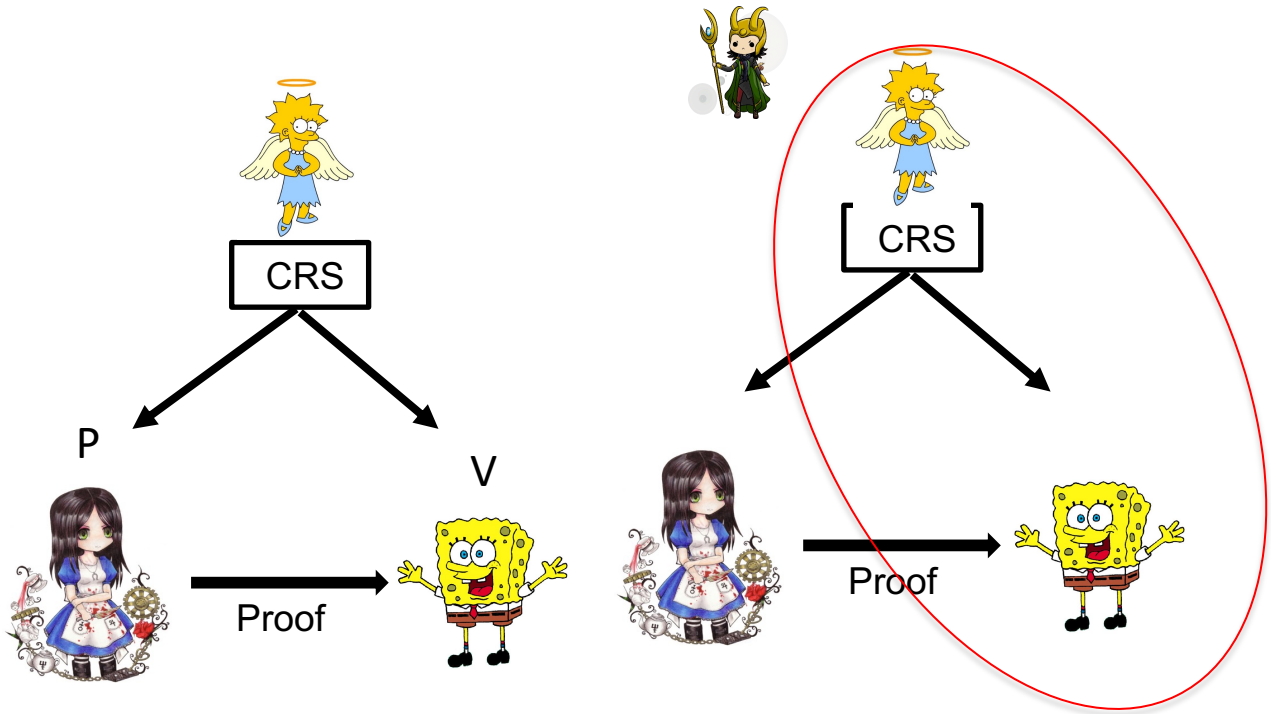

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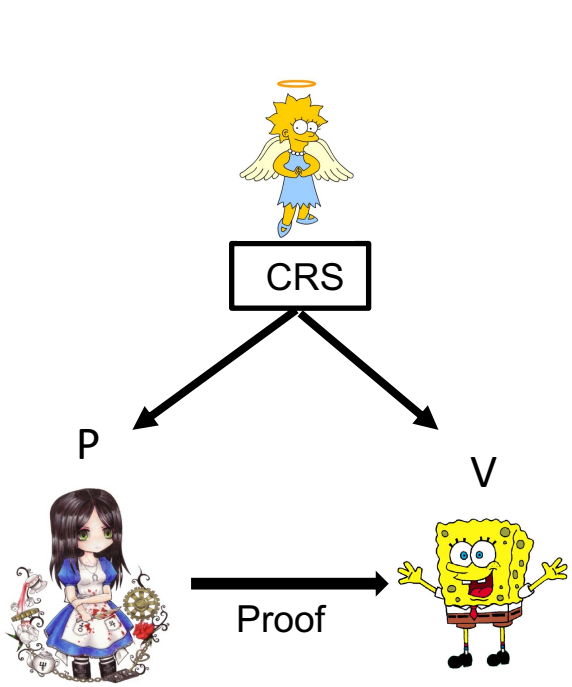

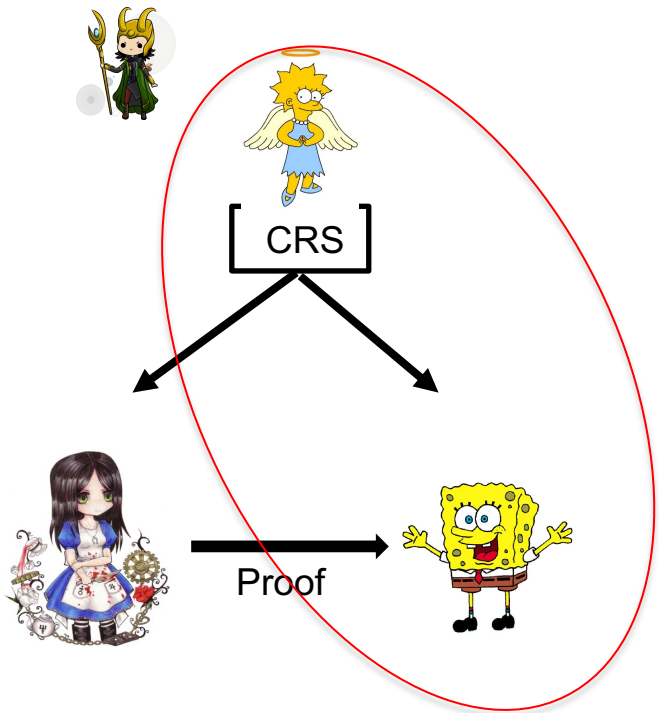

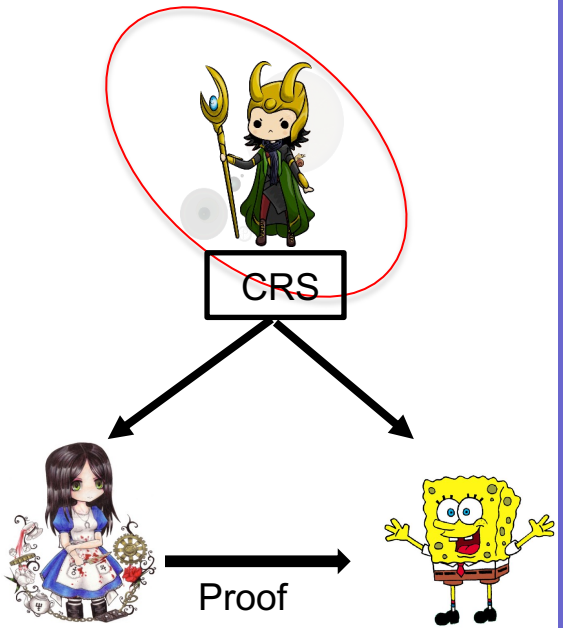

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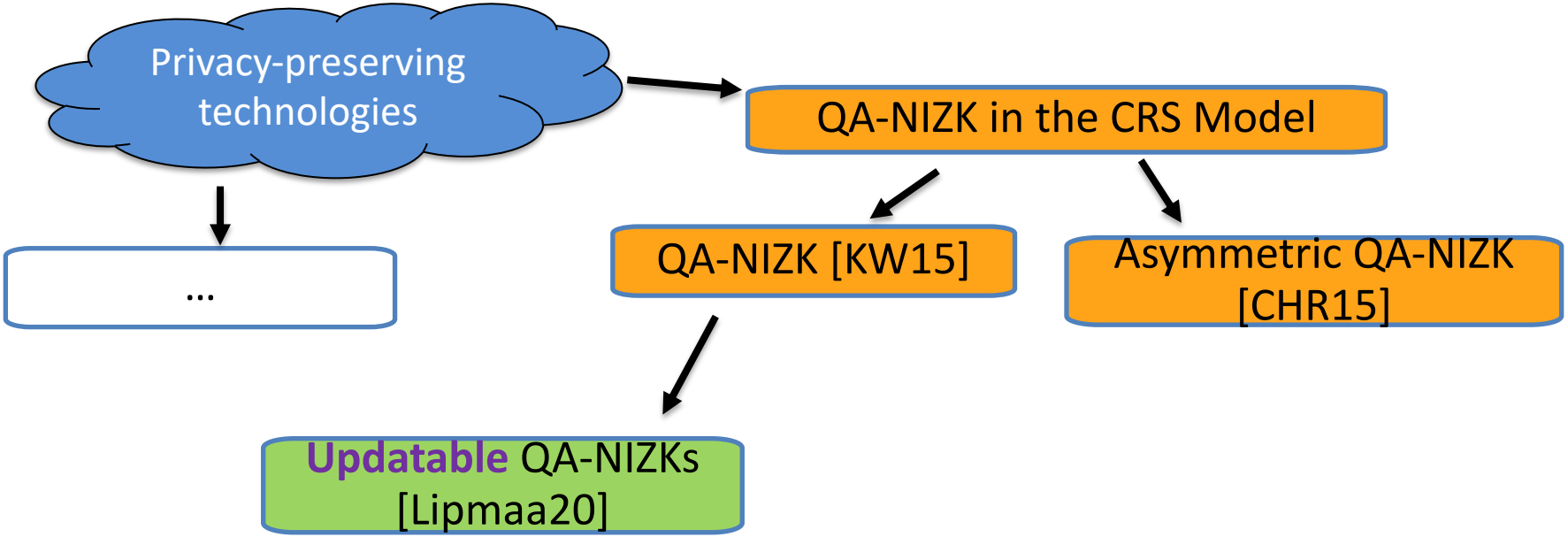
Updatable model
(i.e., [C:GKM+18]…)

State-of-Art of QANIZK in the Updatable setting



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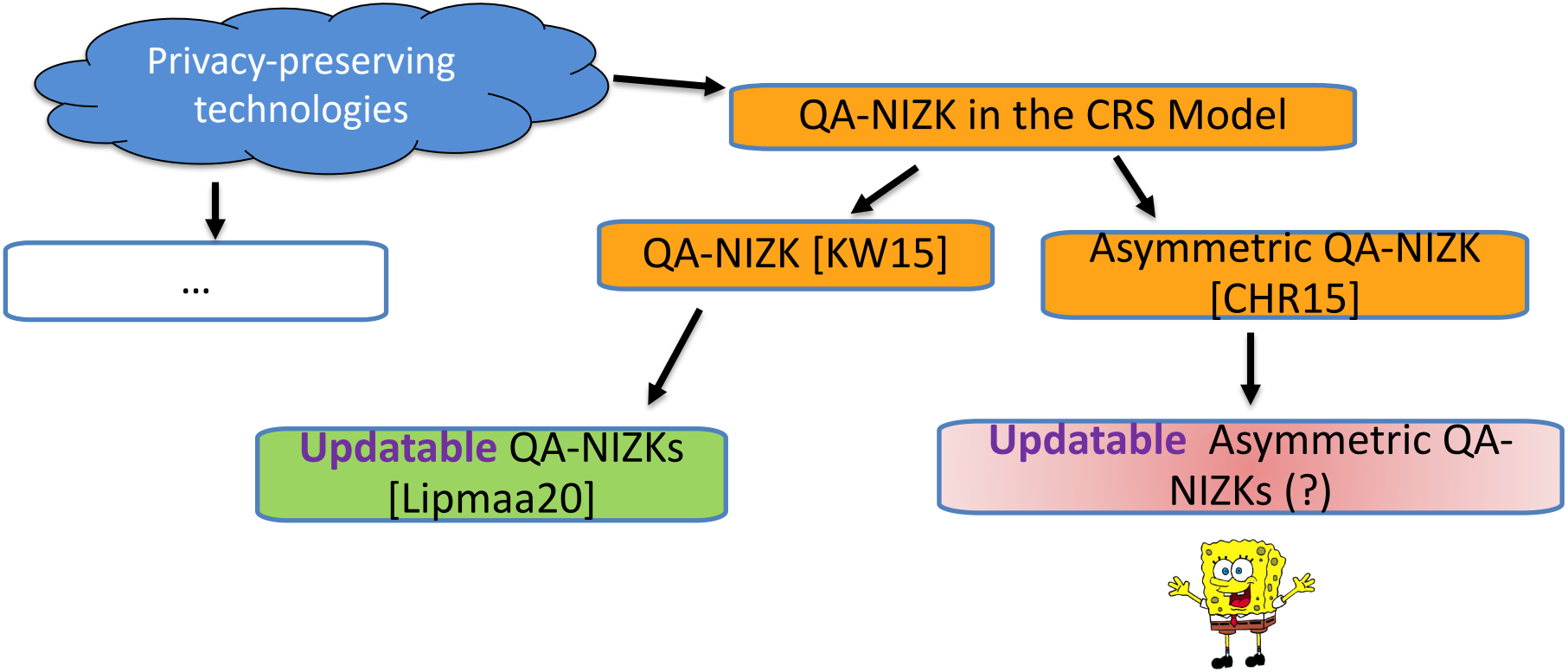


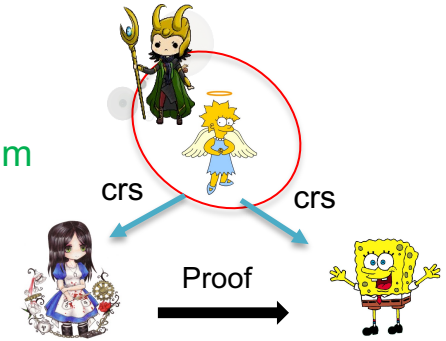
Our Results:

Our main result: Updatable Asymmetric QA-NIZK

- **Zero-knowledge and soundness hold even if CRS creator is not trusted.**

Soundness => verifier does not need to trust CRS – just apply a new Up-crs algorithm and update the CRS to CRS'.

ZK => prover does not need to trust CRS – just apply a new Vcrs algorithm.



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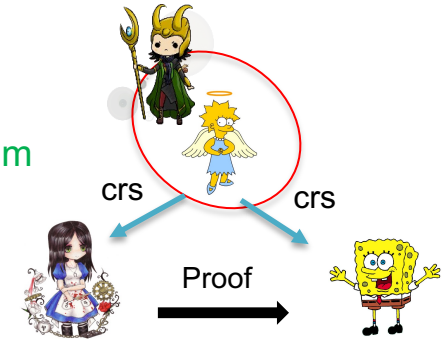

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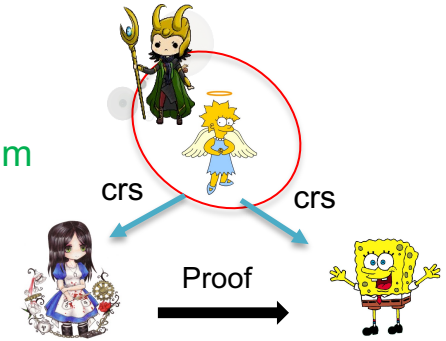

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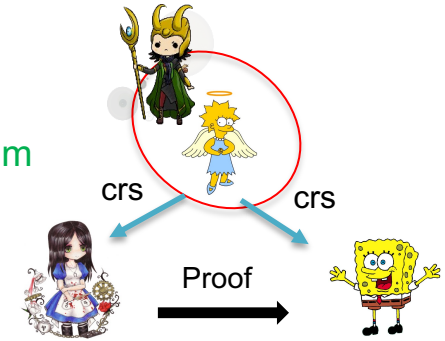

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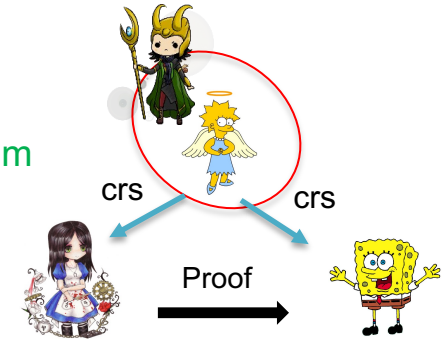

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Proving updatable ZK

If Vcrs($[M]_1$,$[N]_2$, CRS, CRS') = 0: no need to simulate

If Vcrs($[M]_1$,$[N]_2$, CRS, CRS') = 1:

Use extractor Ext to recover tc from CRS' by BDH-KE assumption.

Other results

- Knowledge Sound version of Asymmetric QA-NIZK CHR15 (ASIACRYPT'15) and the updatable Asymmetric QA-NIZK.

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- How to integrate our updatable Knowledge Sound QA-NIZKs (and also Knowledge Sound version of Asymmetric QA-NIZK CHR15) into the LegoSNARK toolbox.
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Open Problems:

- (Sub-ZK) QA-NIZK with Simulation-Sound Extractability


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